

$$\rightarrow \text{like radicals } 2\sqrt{5} + 3\sqrt{5} = 5\sqrt{5}$$

$$2\sqrt{5} + 3\sqrt{5}$$

5. Simplify each expression.

a) $6\sqrt{2} + 3\sqrt{2} = 9\sqrt{2}$

b) $5\sqrt{3} - 9\sqrt{3}$

c) $7\sqrt{32} + 4\sqrt{2}$

d) $7\sqrt{48} + 4\sqrt{3}$

$$7\sqrt{16\sqrt{2}}$$

$$7(4)\sqrt{2} + 4\sqrt{2}$$

$$28\sqrt{2} + 4\sqrt{2} = 32\sqrt{2}$$

e) $3\sqrt[3]{54} + 5\sqrt[3]{16}$

f) $5\sqrt[3]{48} - 2\sqrt[3]{243}$

$$3\sqrt[3]{27\sqrt{2}} + 5\sqrt[3]{8\sqrt{2}}$$

$$5\sqrt[3]{16\sqrt{3}} - 2\sqrt[3]{81\sqrt{3}}$$

$$3(3)\sqrt[3]{2} + 5(2)\sqrt[3]{2} = 19\sqrt[3]{2}$$

$$5(2)\sqrt[3]{3} - 2(3)\sqrt[3]{3} = 4\sqrt[3]{3}$$

g) $7\sqrt{63} - 2\sqrt{28}$

h) $3\sqrt{40} - 8\sqrt{90}$

$$7\sqrt{9\sqrt{7}} - 2\sqrt{4\sqrt{7}}$$

$$7(3)\sqrt{7} - 2(2)\sqrt{7}$$

$$21\sqrt{7} - 4\sqrt{7} = 17\sqrt{7}$$

i) $4\sqrt{12} + 2\sqrt{27} - 3\sqrt{75}$

j) $5\sqrt{18} - 4\sqrt{50} - 2\sqrt{72}$

$$4\sqrt{4\sqrt{3}} + 2\sqrt{9\sqrt{3}} - 3\sqrt{25\sqrt{3}}$$

$$4(2)\sqrt{3} + 2(3)\sqrt{3} - 3(5)\sqrt{3}$$

$$8\sqrt{3} + 6\sqrt{3} - 15\sqrt{3} = -1\sqrt{3} = -\sqrt{3}$$

$$4\sqrt{25\sqrt{2}} + 3\sqrt{4\sqrt{3}} - 5\sqrt{9\sqrt{3}}$$

k) $4\sqrt{50} + 3\sqrt{12} - 5\sqrt{27}$

l) $5\sqrt{72} + 3\sqrt{48} - 4\sqrt{128}$

$$4\sqrt{25\sqrt{2}} + 3\sqrt{4\sqrt{3}} - 5\sqrt{9\sqrt{3}}$$

$$4(5)\sqrt{2} + 3(2)\sqrt{3} - 5(3)\sqrt{3}$$

$$20\sqrt{2} + 6\sqrt{3} - 15\sqrt{3}$$

$$20\sqrt{2} - 9\sqrt{3}$$

m) $\sqrt{162} - \sqrt{50} + \sqrt{75} - \sqrt{108}$

n) $\sqrt{48} + \sqrt{8} - \sqrt{27} - \sqrt{32}$

$$= 9\sqrt{2} - 5\sqrt{2} + 5\sqrt{3} - 6\sqrt{3}$$

$$= 4\sqrt{2} - \sqrt{3}$$

o) $\frac{1}{4}\sqrt{80} + \frac{2}{3}\sqrt{45} - \frac{1}{2}\sqrt{20} = 2\sqrt{5}$

p) $\frac{2}{3}\sqrt{12} - \frac{5}{2}\sqrt{48} + \frac{1}{4}\sqrt{108}$

$$\frac{1}{4}\sqrt{16\sqrt{5}} + \frac{2}{3}\sqrt{9\sqrt{5}} - \frac{1}{2}\sqrt{4\sqrt{5}}$$

$$\frac{1}{4}(4)\sqrt{5} + \frac{2}{3}(6)\sqrt{5} - \frac{1}{2}(2)\sqrt{5} = \sqrt{5} + 2\sqrt{5} - \sqrt{5}$$

q) $5\sqrt[3]{32} + 2\sqrt[3]{8} \cdot \sqrt[3]{4}$

r) $8\sqrt[3]{16} - 10\sqrt[3]{3} \cdot \sqrt[3]{18}$

$$5\sqrt[3]{16\sqrt{2}} + 2\sqrt[3]{32}$$

$$5(2)\sqrt[3]{2} + 2(2)\sqrt[3]{2} = 14\sqrt[3]{2}$$

6. Simplify each expression. All variables represent positive numbers.

a) $\sqrt{3x^2} + \sqrt{12x^2}$

$$\sqrt{3}\sqrt{x^2} + \sqrt{4}\sqrt{3}\sqrt{x^2}$$

$$x\sqrt{3} + 2x\sqrt{3} = \boxed{3x\sqrt{3}}$$

b) $3\sqrt{2x^3} + 5x\sqrt{8x}$

c) $\sqrt{4x^7} - 5x^2\sqrt{x^3} + 3x\sqrt{x^5}$

$$\sqrt{4}\sqrt{x^6}\sqrt{x} - 5x^2\sqrt{x^2}\sqrt{x} + 3x\sqrt{x^4}\sqrt{x}$$

$$\boxed{2x^3}\sqrt{x} - \boxed{5x^2(x)}\sqrt{x} + \boxed{3x(x^2)}\sqrt{x}$$

$$-3x^3\sqrt{x} + 3x^3\sqrt{x} = \text{☺}$$

d) $\sqrt{9x^3} - \sqrt{25x^3} + x\sqrt{16x}$

e) $3\sqrt{125x^2y} + 6x\sqrt{80y}$

f) $5\sqrt{12x} - 3\sqrt{27x}$

g) $5x\sqrt{63y} + 3\sqrt{28x^2y}$

h) $3y\sqrt{24x^2y^2} + 6x\sqrt{54y^3}$

$$3y\sqrt{4}\sqrt{6}\sqrt{x^2}\sqrt{y^2} + 6x\sqrt{9}\sqrt{6}\sqrt{y^2}\sqrt{y}$$

$$\underline{3y(2)\sqrt{6}xy} + \underline{6x(3)\sqrt{6}y^2y}$$

$$6xy^2\sqrt{6} + 18xy\sqrt{6y}$$

i) $\sqrt{\frac{28}{x^2}} + \sqrt{\frac{7}{4x^2}} \quad \frac{\sqrt{4}\sqrt{7}}{\sqrt{x^2}} + \frac{\sqrt{7}}{\sqrt{4}\sqrt{x^2}}$

$$\left(\frac{2}{2}\right)\frac{2\sqrt{7}}{x} + \frac{\sqrt{7}}{2x} = \frac{5\sqrt{7}}{2x} \quad (x \neq 0)$$

j) $\frac{\sqrt{99}}{5x} + \sqrt{\frac{44}{x^2}}$

k) $5\sqrt{8x^2y^3} - 3x\sqrt{32y^3}$

l) $\sqrt{9y+27} + \sqrt{y+3}$

m) $\sqrt{9x-9} + \sqrt{x-1}$

$$\sqrt{9(x-1)} + \sqrt{x-1}$$

$$3\sqrt{x-1} + \sqrt{x-1}$$

$$= 4\sqrt{x-1}$$

n) $2x\sqrt{4x^2z} + 5x\sqrt{9z} - 10x\sqrt{25z}$

o) $\frac{1}{3}\sqrt{x^2} - \frac{5}{3}\sqrt{x^2} + \frac{5}{6}\sqrt{x^2}$

p) $\frac{2}{3}\sqrt{9x^2} + \frac{5}{3}\sqrt{4x^2} + \sqrt{25x^2}$

q) $\sqrt{x^2-6x+9} - \sqrt{x^2-2x+1}$

$$(x-3)\sqrt{(x-3)} - (x-1)\sqrt{(x-1)}$$

$$\sqrt{(x-3)^2} - \sqrt{(x-1)^2} \quad x-3 - (x-1) = -2$$

r) $\sqrt{x^2+4x+4} - \sqrt{x^2+10x+25}$

7. Simplify each expression. All variables represent positive numbers.

a) $\sqrt[3]{27} - 4\sqrt[3]{8}$

b) $3\sqrt[3]{6} + 2\sqrt[3]{48}$

c) $\sqrt[3]{x^4} - \sqrt[3]{x^7}$

d) $x\sqrt[3]{8x^5} + \sqrt[3]{27x^8}$
 $x\sqrt[3]{8}\sqrt[3]{x^3}\sqrt[3]{x^2} + \sqrt[3]{27}\sqrt[3]{x^6}\sqrt[3]{x^2}$
 $x(2)x\sqrt[3]{x^2} + 3x^2\sqrt[3]{x^2} = 5x^2\sqrt[3]{x^2}$

e) $6\sqrt[3]{8x^2} - 2\sqrt[3]{27x^2}$

f) $4\sqrt[3]{27x^2} + 6\sqrt[3]{8x^2}$

g) $3\sqrt[4]{x^5} - 2x\sqrt[4]{16x}$

h) $2\sqrt[4]{6x^7} - x\sqrt[4]{96x^3}$

i) $-4\sqrt[3]{256x^4} - 2x\sqrt[3]{32x}$

j) $\frac{5}{2}\sqrt[3]{16x^4y^5} + xy\sqrt[3]{54xy^2}$

k) $-\frac{\sqrt[3]{2x^4}}{9} + \sqrt[3]{\frac{250x^4}{27}}$

l) $\frac{\sqrt[3]{x^5}}{8} + \frac{5x\sqrt[3]{x^2}}{4}$

m) $3\sqrt[4]{16x} - 5\sqrt[4]{x^5} + x\sqrt[4]{81x}$

n) $4\sqrt[4]{32x^5} - 2x\sqrt[4]{2x} + 7\sqrt[4]{x^5}$

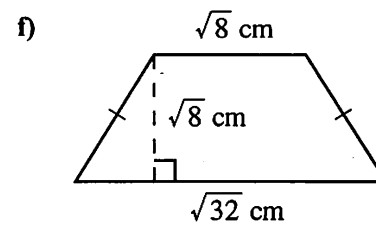
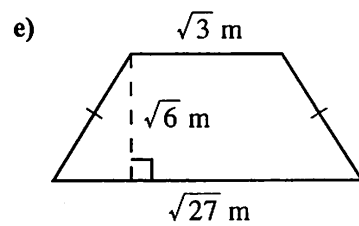
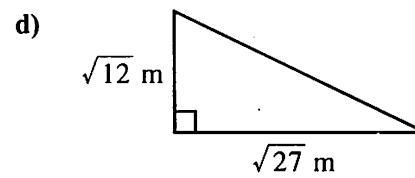
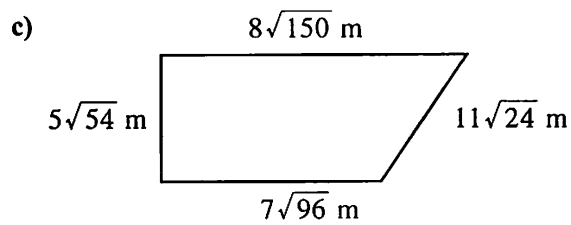
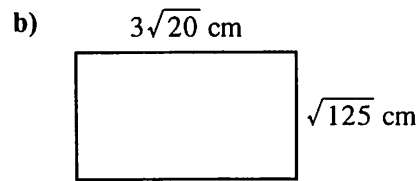
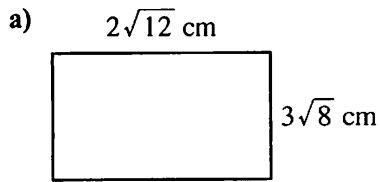
o) $\sqrt{20x^3y} - \sqrt{45x^5y^3} + \sqrt{80x^7y^5}$

p) $x\sqrt{48xy^2} + y\sqrt{27x^3} - \sqrt{75x^3y^2}$

q) $\sqrt{8x^2y^3} + xy\sqrt{64y^2}$

r) $\sqrt[3]{8x^3y} + \sqrt[3]{64x^6y^2}$

8. Find the perimeter.



9. Explain why $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$. Give examples.

1.5 Exercise Set

1. Fill in the blank to make the statement correct.

a) Changing a radical in the denominator of a fraction to a rational number is called _____.

b) In the fraction $\frac{3}{\sqrt{2}}$, the 3 is called the _____ and the $\sqrt{2}$ is called the _____.c) $2 + \sqrt{5}$ is the _____ of $2 - \sqrt{5}$.d) To rationalize the denominator of $\frac{1}{\sqrt{x}}$, the _____ and _____ are multiplied by _____.e) To rationalize the denominator of $\frac{1}{\sqrt{2} + 1}$, the _____ and _____ are multiplied by _____.

2. Multiply. All variables represent positive numbers. Express answers in simplest radical form.

a) $(\sqrt{5})^2$ _____ b) $(-5\sqrt{2})^2$ _____

c) $\sqrt{2} \cdot \sqrt{8}$ _____ d) $\sqrt{3} \cdot \sqrt{27}$ _____

e) $(-\sqrt[3]{5})^3$ _____ f) $\sqrt{6} \cdot \sqrt{8}$ _____

g) $\sqrt{a^4} \cdot \sqrt{a^6}$ _____ h) $(4\sqrt{x})(-3\sqrt{x})$ _____

i) $\sqrt{2x} \cdot \sqrt{6x^3}$ _____ j) $(-3\sqrt{6})(4\sqrt{3})$ _____

k) $-3\sqrt{2x} \cdot \sqrt{12y}$ _____ l) $(-2\sqrt[3]{4x^2})(-4\sqrt[3]{6x^2})$ _____

m) $(-2x\sqrt{xy^2})(6y\sqrt{x^2y})$ _____ n) $(4xy\sqrt[3]{x^2y})(3xy\sqrt[3]{xy^2})$ _____

$$3(x-2) = 3x-6$$

$$(x-2)(x-3)$$

$$\sqrt{6} \cdot \sqrt{6} = 6$$

$$\sqrt{u} \cdot \sqrt{u} = u$$

Difference of \square 's
 $(a-b)(a+b)$
 $a^2 - b^2$

3. Find each product and simplify.

$$x^2 - 3x - 2x + 6$$

$$x^2 - 5x + 6$$

b) $2\sqrt{3}(7\sqrt{5} - 5\sqrt{3})$

a) $\sqrt{5}(\sqrt{8} + \sqrt{32})$

$$\sqrt{5}(2\sqrt{2} + 4\sqrt{2})$$

$$2\sqrt{10} + 4\sqrt{10} = 6\sqrt{10}$$

c) $2\sqrt[3]{5}(3\sqrt[3]{3} - \sqrt[3]{25})$

d) $2\sqrt[3]{2}(5\sqrt[3]{4} + 3\sqrt[3]{2})$

e) $(1 - \sqrt{2})(1 - \sqrt{2})$

f) $(1 - \sqrt{2})(1 + \sqrt{2})$

g) $(4 + \sqrt{3})(4 + \sqrt{3})$

h) $(4 + \sqrt{3})(4 - \sqrt{3})$

$$16 + 4\sqrt{3} + 4\sqrt{3} + 3$$

$$19 + 8\sqrt{3}$$

i) $(\sqrt{5} - \sqrt{2})(\sqrt{5} - \sqrt{2})$

j) $(\sqrt{5} + \sqrt{2})^2$

$$5 - \sqrt{10} - \sqrt{10} + 2$$

$$7 - 2\sqrt{10}$$

k) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$

l) $\left(\frac{5 - \sqrt{50}}{5}\right)^2$

$$\frac{5 - 5\sqrt{2}}{5} \cdot \frac{5 + 5\sqrt{2}}{5} = (1 - \sqrt{2})(1 + \sqrt{2})$$

n) $(\sqrt[3]{4} + 2)(\sqrt[3]{2} - 1)$

$$\frac{5 - \sqrt{50}}{5} \cdot \frac{5 + \sqrt{50}}{5} = (1 - \sqrt{2})(1 + \sqrt{2})$$

$$1 - 2 = -1$$

o) $(\sqrt[3]{4} - 1)(\sqrt[3]{4} + 1)$

p) $(\sqrt[3]{4} - 1)^2$

$$\sqrt[3]{16} - 1$$

$$2 - 1 = 1$$

q) $(\sqrt[3]{4} - 1)(\sqrt[3]{16} + \sqrt[3]{4} + 1)$

r) $(\sqrt[3]{2} + \sqrt[3]{3})(\sqrt[3]{4} - \sqrt[3]{6} + \sqrt[3]{9})$

$$\sqrt[3]{64} + \sqrt[3]{16} + \sqrt[3]{4} - \sqrt[3]{16} - \sqrt[3]{4} - 1$$

$$4 - 1 = 3$$

4. Find each product and simplify. Assume all variables are positive.

a) $\sqrt{2x}(\sqrt{2} - \sqrt{x})$
 $\sqrt{4x} - \sqrt{2x^2}$
 $2\sqrt{x} - x\sqrt{2}$

b) $\sqrt{7y}(\sqrt{y} + \sqrt{7})$

c) $(2x - \sqrt{3})(2x + \sqrt{3})$
 $4x^2 - 3$

d) $(2x - \sqrt{3})(2x - \sqrt{3})$
 $4x^2 - 2x\sqrt{3} - 2x\sqrt{3} + 3$
 $4x^2 - 4x\sqrt{3} + 3$

e) $(\sqrt{x+2})^2$

f) $(\sqrt{x} + 2)^2$

g) $(\sqrt{x-3} - 4)^2$

h) $(\sqrt{x-3} - 4)(\sqrt{x-3} + 4)$

i) $(3\sqrt{x} + \sqrt{y})^2$

j) $(\sqrt{x} + 3\sqrt{6})(\sqrt{x} - 3\sqrt{6})$

k) $(\sqrt{x} - 2)^2 - (\sqrt{x} + 2)^2$

l) $(\sqrt{x} - y)(\sqrt{x} + y)$

m) $(\sqrt{x+2} + 3)(\sqrt{x+2} - 3)$

n) $(\sqrt{x+2} + 3)^2$

o) $(\sqrt{y+1} + \sqrt{y-1})(\sqrt{y+1} - \sqrt{y-1})$
 $y+1 - (y-1) = 2$

p) $(\sqrt{2x} + \sqrt{y})(\sqrt{2x} - 5\sqrt{y})$

q) $(\sqrt{x+1})^2 - (\sqrt{x} + 1)^2$
 $x+1 - (\sqrt{x}+1)(\sqrt{x}+1)$
 $x+1 - (x+2\sqrt{x}+1)$
 $= -2\sqrt{x}$

r) $(\sqrt[3]{x} + 1)(\sqrt[3]{x^2} - \sqrt[3]{x} + 1)$

5. Perform the indicated operation and simplify. Assume all variables represent positive real numbers.

a) $\sqrt{x} \cdot \sqrt[3]{x}$

b) $\frac{\sqrt{x}}{\sqrt[3]{x}}$

c) $\sqrt{x^3} \cdot \sqrt[5]{x^2}$

d) $\frac{\sqrt{x^3}}{\sqrt[5]{x^2}}$

e) $\sqrt[4]{a^3} \cdot \sqrt[3]{a^2}$

f) $\frac{\sqrt{ab^3}}{\sqrt[5]{a^2b^3}}$

g) $\sqrt{16x^3y^3} \cdot \sqrt[3]{8xy^2}$

h) $\frac{\sqrt[4]{x^2y^3}}{\sqrt{xy}}$

i) $\sqrt{(3-2x)^3} \cdot \sqrt[3]{(3-2x)^2}$

j) $\frac{\sqrt[3]{(1-x)^2}}{\sqrt[4]{1-x}}$

k) $\sqrt[3]{2x-1} \cdot \sqrt{2x-1}$

l) $\frac{\sqrt{(2x-1)^4}}{\sqrt[4]{(2x-1)^3}}$

m) $\sqrt{x+3} \cdot \sqrt[4]{(x+3)^2}$

n) $\frac{\sqrt[3]{(2-x)^4}}{\sqrt[4]{(2-x)^3}}$

o) $\sqrt{x} \cdot \sqrt[3]{x^2} \cdot \sqrt[4]{x^3}$

p) $\frac{\sqrt{x} \cdot \sqrt[3]{x^2}}{\sqrt[4]{x^3}}$

6. Determine what the fraction must be multiplied by to rationalize the denominator.

a) $\frac{1}{\sqrt{x-2}}$

b) $\frac{1}{\sqrt{x}-\sqrt{2}}$

c) $\frac{1}{\sqrt[3]{x-2}}$

d) $\frac{\sqrt{x}-2}{\sqrt{x}}$

e) $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}+\sqrt{3}}$

f) $\frac{\sqrt{x}-1}{\sqrt{x}+3}$

g) $\frac{\sqrt{x}}{\sqrt{x}-\sqrt{y}}$

h) $\frac{\sqrt{y}}{\sqrt{x}+\sqrt{y}}$

7. Simplify, if possible.

a) $\sqrt{2} + \sqrt{5}$

b) $\sqrt{2} \cdot \sqrt{5}$

c) $\sqrt{6} - \sqrt{3}$

d) $\frac{\sqrt{6}}{\sqrt{3}}$

e) $\sqrt{3} - 2\sqrt{3}$

f) $\sqrt{3} \cdot 2\sqrt{3}$

g) $\frac{\sqrt{3}}{4\sqrt{3}}$

h) $\frac{\sqrt{3}}{4\sqrt{2}}$


i) $\frac{1}{\sqrt[3]{3}}$

j) $\frac{1}{\sqrt[3]{2}}$

8. Mentally find each product. Assume all variables are positive.

a) $(\sqrt{5} + 2)(\sqrt{5} - 2)$
 $5 - 4 = 1$

b) $(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})$

 conjugate

c) $(4 + \sqrt{3})(4 - \sqrt{3})$
 $16 - 3 = 13$

d) $(\sqrt{x} - y)(\sqrt{x} + y)$

e) $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})$
 $x - y$

f) $(\sqrt{x+1} - 1)(\sqrt{x+1} + 1)$

g) $(\sqrt{x+2} - \sqrt{y+2})(\sqrt{x+2} + \sqrt{y+2})$

h) $(x - \sqrt{x-1})(x + \sqrt{x-1})$

i) $(\sqrt{1-x} - \sqrt{x-1})(\sqrt{1-x} + \sqrt{x-1})$

j) $(\sqrt{2x+3} + \sqrt{2x-3})(\sqrt{2x+3} - \sqrt{2x-3})$
 $2x+3 - (2x-3) = 6$

9. Rationalize the denominator. Write the quotient in lowest terms. Assume all variables are positive.

a) $\frac{1}{\sqrt{2}} \left(\frac{\sqrt{2}}{\sqrt{2}} \right) = \frac{\sqrt{2}}{2}$

b) $\frac{1}{\sqrt[3]{2}}$

c) $\frac{3 + \sqrt{2}}{\sqrt{2}} \left(\frac{\sqrt{2}}{\sqrt{2}} \right) = \frac{3\sqrt{2} + 2}{2}$

d) $\frac{5 - \sqrt{2}}{\sqrt{3}}$

e) $\frac{1}{3 + \sqrt{2}} \left(\frac{3 - \sqrt{2}}{3 - \sqrt{2}} \right) = \frac{3 - \sqrt{2}}{7}$
 $9 - 2$

f) $\frac{1}{3 - \sqrt{2}}$

g) $\frac{\sqrt{12}}{\sqrt{3} + 1} \left(\frac{\sqrt{3} - 1}{\sqrt{3} - 1} \right) = \frac{\sqrt{36} - \sqrt{12}}{2} = \frac{6 - 2\sqrt{3}}{2} = 3 - \sqrt{3}$

h) $\frac{\sqrt{18}}{\sqrt{2} - 1}$

i) $\frac{3 + \sqrt{2}}{1 + \sqrt{2}}$

j) $\frac{\sqrt{5}}{\sqrt{2} - \sqrt{3}}$

k) $\frac{1}{\sqrt{x} - \sqrt{y}}$

l) $\frac{\sqrt{a} + b}{\sqrt{a} - b}$